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| 6. AUTHORS<br>Michael J Biercuk, Lorenza Viola, Amir Yacoby | 5d. PROJECT NUMBER |
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14. ABSTRACT

Abstract:
This project aimed to address the most significant challenge facing the development of large-scale quantum computers: suppressing hardware error. Through this work we focused on the realization of a robust, high-fidelity Quantum Control Toolkit that is fully transportable between different quantum computing technology platforms. The techniques encapsulated herein were designed to provide qubit robustness to decoherence during memory (dynamical decoupling - DD) and increase the fidelity of qubit operations undertaken in the presence of random

15. SUBJECT TERMS

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| a. REPORT | b. ABSTRACT | c. THIS PAGE | | | Michael Biercuk |
| UU | UU | UU | UU | | 19b. TELEPHONE NUMBER<br>+61-290-3653 |

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## Report Title

Final Report: Precision Quantum Control and Error-Suppressing Quantum Firmware for Robust Quantum Computing

## ABSTRACT

Abstract:
This project aimed to address the most significant challenge facing the development of large-scale quantum computers: suppressing hardware error.  Through this work we focused on the realization of a robust, high-fidelity Quantum Control Toolkit that is fully transportable between different quantum computing technology platforms. The techniques encapsulated herein were designed to provide qubit robustness to decoherence during memory (dynamical decoupling – DD), and increase the fidelity of qubit operations undertaken in the presence of random environmental noise and control imperfections (dynamically corrected gates – DCGs, and composite pulses – CPs).

This project has seen the demonstration of significant hardware-based improvements in quantum control fidelity along with the full experimental validation of an engineering-inspired filter-transfer function framework for quantum control.

**Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:**

**(a) Papers published in peer-reviewed journals (N/A for none)**

| Received | | Paper |
|---|---|---|
| 08/22/2011 | 2.00 | A C Doherty, M J Biercuk, H Uys. Dynamical decoupling sequence construction as a filter-design problem, Journal of Physics B: Atomic, Molecular and Optical Physics, (08 2011): 154002. doi: 10.1088/0953-4075/44/15/154002 |
| 08/23/2011 | 1.00 | Michael Biercuk, Hendrik Bluhm. Phenomenological study of decoherence in solid-state spin qubits due to nuclear spin diffusion, Physical Review B, (6 2011): 235316. doi: 10.1103/PhysRevB.83.235316 |
| 08/31/2012 | 5.00 | M. D. Shulman, O. E. Dial, S. P. Harvey, H. Bluhm, V. Umansky, A. Yacoby. Demonstration of Entanglement of Electrostatically Coupled Singlet-Triplet Qubits, Science, (04 2012): 0. doi: 10.1126/science.1217692 |
| 08/31/2012 | 6.00 | David Hayes, Kaveh Khodjasteh, Lorenza Viola, Michael Biercuk. Reducing sequencing complexity in dynamical quantum error suppression by Walsh modulation, Physical Review A, (12 2011): 0. doi: 10.1103/PhysRevA.84.062323 |
| 09/21/2014 | 24.00 | C. Kabytayev, T. J. Green, K. Khodjasteh, M.J. Biercuk, L. Viola, K. R. Brown . “Robustness of composite pulses to time-dependent control noise,” , PHYSICAL REVIEW A .89,012316 (2014), (07 2014): 0. doi: |
| 09/21/2014 | 30.00 | Todd Green, Hermann Uys, M.J. Biercuk. “High-order noise filtering in nontrivial quantum logic gates,” , Phys Rev Lett 109,020501 (2012), (07 2012): 0. doi: |
| 09/21/2014 | 29.00 | Kaveh Khodjasteh, Hendrik Bluhm, Lorenza Viola. “Automated synthesis of dynamically corrected quantum gates,”, PHYSICAL REVIEW A 86,042329, (10 2012): 0. doi: |
| 09/21/2014 | 27.00 | O. E. Dial, M. D. Shulman, S. P. Harvey, H. Bluhm, V. Umansky, A. Yacoby. “Electrometry Using Coherent Exchange Oscillations in a Singlet-Triplet-Qubit,” , Phys Rev Lett 110,146804 (2013), (04 2013): 0. doi: |
| 09/21/2014 | 26.00 | Todd J. Green, Jarrah Sastrawan, Hermann Uys, M.J. Biercuk. “Arbitrary quantum control of qubits in the presence of universal noise,” , New Journal of Physics, (09 2013): 0. doi: |
| 09/21/2014 | 25.00 | A. Soare, H. Ball, D. Hayes, X. Zhen, M. C. Jarratt, H. Uys, M.J. Biercuk. “Experimental bath engineering for quantitative studies of quantum control,” , PHYSICAL REVIEW A 89,042329 (2014), (04 2014): 0. doi: |
| 09/22/2014 | 21.00 | M.W. Lee, M.C. Jarratt, C. Marciniak, M.J. Biercuk. “Frequency Stabilization of a 369 nm Diode Laser by Nonlinear Spectroscopy of Ytterbium Ions in a Discharge,” , Opt. Express, (03 2014): 0. doi: |
| 09/23/2014 | 16.00 | L. Viola, F. Ticozzi . “Quantum resources for purification and cooling: fundamental limits and opportunities,”, , Scientific Reports 4,5192 (2014), (06 2014): 0. doi: |
| 09/23/2014 | 17.00 | Adam D. Bookatz, Pawel Wocjan, Lorenza Viola. “Hamiltonian quantum simulation with bounded-strength controls,”, New Journal of Physics, (04 2014): 0. doi: |

09/23/2014 18.00 D.Hayes, S.T. Flammia, M.J. Biercuk. “Programmable quantum simulation by dynamic Hamiltonian engineering,”,
New Journal of Physics, (08 2014): 0. doi:

09/23/2014 11.00 K.Khodjasteh, J.Sastrawan, D. Hayes, T.J.Green, M.J. Biercuk, L. Viola. Designing a practical high-fidelity long-time quantum memory,
Nature Communications, (06 2013): 0. doi:

**TOTAL:** **15**

**Number of Papers published in peer-reviewed journals:**

---

## (b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

**TOTAL:**

**Number of Papers published in non peer-reviewed journals:** 5

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## (c) Presentations

Biercuk:

• Noise Filtering in Quantum Systems Noise filtering in quantum systems –
China Workshop on Quantum Control Brisbane Aust Wednesday, 1 October, 2014
• Noise Filtering in Quantum Systems Noise Filtering in Quantum Systems –
PRACQSYS Cambridge UK Thursday, 7 August, 2014
• Control in Quantum Coherent Systems Control in Quantum Coherent Systems –
MIT University USA Thursday, 31 July, 2014
• Control in Quantum Coherent Systems Control in Quantum Coherent Systems –
Princeton University USA Thursday, 24 July, 2014
• Engineering the Quantum Future Invitational talk at the Wesley College High-Table Dinner - Wesley College Monday, 28 April, 2014
• Contributing Student Talk - American Physical Society March Meeting Contributing Student Talk, Alex Soare - American Physical Society Conference in Denver Colorado - March 2014
Michael Biercuk, Alexander Soare Colorado United States of America Sunday, 2 March, 2014
• Invited Seminar, Building Noise Filters by Quantum Control, Stanford University - California United States of America Friday, 24 January, 2014
• Programmable quantum simulation with trapped ions University of Sussex/ IQSIM 13, Dec 2013
"Precision Quantum Control with Trapped 171Yb+ Ions," EQuS Workshop, Dec 2013
• Control in quantum coherent systems, The University of Pennsylvania Electrical and Systems Engineering Colloquium, Philadelphia, USA, Nov 2013
"Control Engineering in Quantum Coherent Systems," ANU Electrical Engineering Colloquium, Sept 2013
"Controlling quantum systems with photons," Univ. Adelaide IPAS Seminar, Aug 2013
"Precision Quantum Control with Trapped 171Yb+ Ions," Soare et al., APS March 2013
"Quantum logic gates by Walsh modulation," Ball et al., APS March 2013
"Robustness of composite pulse sequences to time- dependent noise", Kabytayev et al, APS March 2013
"From Quantum Control to Quantum Simulation with trapped ions,"

- University of Pennsylvania Dept of Physics, Apr 2013
- Princeton University, MITRE seminar, Apr 2013
- KITP Workshop on Quantum Control, Feb 2013
- AIP Congress, Dec 2012
- Tsinghua University, Oct 2012
- Australia-China workshop on quantum control, Oct 2012
- ANU CQC2T Seminar, June 2012

"Controlling trapped atoms with light," Quantum Rottnest, Sept 2012
"Quantum control to Quantum Simulation," EQuS Workshop Feb 2012
"Quantum Firmware," QEC 11 Invited Presentation, Nov 2011
"Quantum Firmware," Google Australia, Invited Seminar, Oct 2011

Lorenza Viola:
"Controlling open quantum systems: From dissipation-protected to dissipation-driven quantum engineering", Colloquium Talks Series, Max-Planck-Institute of Quantum Optics, Garching, Germany, June 11, 2013
"Advances in quantum Hamiltonian engineering", Interdisciplinary Quantum Information Science & Engineering Seminars, MIT, Cambridge, March 13, 2014
"Steady-state entanglement engineering with quasi-local dissipation", Center for Quantum Information and Control Seminars, University of New Mexico, May 8, 2014
Colloquium Talks Series, Max-Planck-Institute of Quantum Optics, Garching, Germany, June 11, 2013
"Advances in dynamical quantum error suppression", Kavli Institute for Theoretical Physics, Santa Barbara, February 19, 2013
"Dynamical quantum error correction: achievements and prospects", March Meeting of the American Physical Society, Baltimore, March 20, 2013

"Controlling open quantum systems: From dissipation-protected to dissipation-driven quantum engineering",

- Quantum Sciences & Chemistry Seminar Series, Harvard, Cambridge, October 12, 2012
- Quantum Lunch Seminar, Los Alamos National Laboratory,November 1, 2012
- Colloquium Talks Series, Max-Planck-Institute of Quantum Optics, Garching, Germany, June 11, 2013

"Advances in dynamical quantum error suppression", Kavli Institute for Theoretical Physics, Santa Barbara, February 19, 2013
"Dynamical quantum error correction: achievements and prospects", March Meeting of the American Physical Society, Baltimore, March 20, 2013
"Dynamically corrected gates" QEC11
"Dynamically corrected gates "Canadian Summer School on Quantum Information, Waterloo, June 2012.
K. Khodjasteh: "Smallest errors achievable by dynamical decoupling (and How to Maintain Them)" in Quantum Error Correction 2011. invited presentation
2. "Upper bounds on coherence preservation in dynamical decoupling" in American Physics Society's March Meeting 2012,. contributed presentation

A. Yacoby:

Princeton colloquium April 4 2013
Das Sarma 60th festivities March 21 2013
Colloquium BU Feb. 19 2013
AAAS invited talk Feb. 15 2013
ICPS plenary talk Aug. 2012
Invited talks at the Windsor School Aug. 2012
Karles Invitational Symposium Aug, 2012
QIPC 2011. September 2011, Zurich.
Colloquium, University of Waterloo, October 2011.
Fermi School, Varenna. June 2012.

**Number of Presentations:** 50.00

## Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

**TOTAL:**

**Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):**

## Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

**TOTAL:**

**Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):**

## (d) Manuscripts

| Received | | Paper |
|---|---|---|
| 08/28/2013 | 9.00 | . Arbitrary quantum control of qubits in the presence of universal noise, ( ) |
| 08/31/2012 | 8.00 | Kaveh Khodjasteh, Hendrik Bluhm, Lorenza Viola. Automated Synthesis of Dynamically Corrected Quantum Gates, arXiv:1205.0217v2 (07 2012) |
| 08/31/2012 | 7.00 | Kaveh Khodjasteh , Michael J. Biercuk, Lorenza Viola. Long-time Low-latency Quantum Memory by Dynamical Decoupling, arXiv:1206.6087v1 (06 2012) |
| 09/21/2014 | 12.00 | L. Viola, G. A. Paz-Silva . A General Transfer-Function Approach to Noise Filtering in Open-Loop Quantum Control, ArXiv1408.3836(2014) (08 2014) |
| 09/21/2014 | 15.00 | M. D. Shulman, S. P. Harvey, J.M. Nichol, S.D. Bartlett, A.C. Doherty, V. Umansky, A. Yacoby. "Suppressing qubit dephasing using real-time Hamiltonian estimation. , ArXiv 1405.0485 (2014) (08 2014) |
| 09/21/2014 | 13.00 | T. J. Green , M. J. Biercuk. “Phase-modulated decoupling and error suppression in qubit-oscillator systems,” , arXiv:1408.2749 (2014) (08 2014) |
| 09/24/2014 | 14.00 | A.Soare, H. Bal, D. Hayes, M. C. Jarratt, J.J. McLoughlin, X. Zhen, T.J. Green, M.J. Biercuk. “Experimental noise filtering by quantum control”, arXiv:1404.0520 (2014) (08 2014) |

**TOTAL:** **7**

**Number of Manuscripts:**

## Books

| Received | Book |
|---|---|

**TOTAL:**

| Received | Book Chapter |
|---|---|

**TOTAL:**

## Patents Submitted

1. Australian provisional patent application (2013903715), D. Hayes, S. Flammia, M.J. Biercuk, "Programmable quantum simulation"
2. International Patent Application (PCT/AU2013/000649) D. Hayes, K. Khodjasteh L. Viola, M.J. Biercuk, "Long-time low-latency quantum memory by dynamical decoupling."
3. Australian Provisional Patent Application (SPEP - 15336901): D. Hayes, K. Khodjasteh L. Viola, M.J. Biercuk, "Reducing sequencing complexity in dynamical quantum error suppression by Walsh modulation"

## Patents Awarded

## Awards

1. M. J. Biercuk, Shell Australian Innovation Challenge (Shortlist), "Quantum Firmware" (2014)
2. M. J. Biercuk, Shell Australian Innovation Challenge (Finalist), "Quantum Firmware" (2013)
3. M. J. Biercuk, Invitee, Founders Fund F50 technology conference (2013)
4. M. J. Biercuk, Sydney Morning Herald Top 100 Most Influential People (2012).
5. M. J. Biercuk, Australian Museum Eureka Prize for Innovation in Computer Science (Finalist), "Quantum Firmware" (2012)

## Graduate Students

| NAME | PERCENT_SUPPORTED | Discipline |
|---|---|---|
| T. J. Green | 0.00 | |
| **FTE Equivalent:** | **0.00** | |
| **Total Number:** | **1** | |

## Names of Post Doctorates

| NAME | PERCENT_SUPPORTED |
|---|---|
| M.W. Lee | 0.00 |
| D. Hayes | 0.00 |
| T. McRae | 0.00 |
| G. Paz-Silva | 0.00 |
| K. Khodjasteh | 0.00 |
| S.W. Lee | 0.00 |
| **FTE Equivalent:** | **0.00** |
| **Total Number:** | **6** |

## Names of Faculty Supported

| NAME | PERCENT_SUPPORTED | National Academy Member |
|---|---|---|
| Lorenza Viola | 0.00 | |
| **FTE Equivalent:** | **0.00** | |
| **Total Number:** | **1** | |

## Names of Under Graduate students supported

| NAME | PERCENT_SUPPORTED |
|---|---|
| **FTE Equivalent:** | |
| **Total Number:** | |

## Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: ...... 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

## Names of Personnel receiving masters degrees

| NAME |
|---|
| **Total Number:** |

## Names of personnel receiving PHDs

| NAME |
|---|
| **Total Number:** |

## Names of other research staff

| NAME | PERCENT_SUPPORTED |
|---|---|
| **FTE Equivalent:** | |
| **Total Number:** | |

## Sub Contractors (DD882)

# Inventions (DD882)

5 **Long-time low-latency quantum membory by dynamical decoupling**

Patent Filed in US? (5d-1) N
Patent Filed in Foreign Countries? (5d-2) Y
Was the assignment forwarded to the contracting officer? (5e) Y
Foreign Countries of application (5g-2): Australia

5a: D. Hayes, K. Khodjasteh L. Viola, M.J. Biercuk

5f-1a: University of Sydney

5f-c: A28 Physics Road
Sydney NS 2006

5 **Programmable quantum simulation**

Patent Filed in US? (5d-1) N
Patent Filed in Foreign Countries? (5d-2) Y
Was the assignment forwarded to the contracting officer? (5e) Y
Foreign Countries of application (5g-2): Australia

5a: D. Hayes, S. Flammia, M.J. Biercuk,

5f-1a: University of Sydney

5f-c: A28 Physics Road
Sydney NS 2006

5 **Recucing sequencing complexity in dynamical quantum error suppression by Walsh modulation**

Patent Filed in US? (5d-1) N
Patent Filed in Foreign Countries? (5d-2) Y
Was the assignment forwarded to the contracting officer? (5e) Y
Foreign Countries of application (5g-2): Australia

5a: D. Hayes, K. Khodjasteh L. Viola, M.J. Biercuk,

5f-1a: University of Sydney

5f-c: A28 Physics Road
Sydney NS 2006

## Scientific Progress

## Technology Transfer

In 2013 the University of Sydney established a formal partnership with Lockheed Martin corporation supporting research efforts in quantum control. Lockheed has invested in research related to this agreement with a hope that future technology spinoffs may be developed.

**Michael Biercuk Scientific Progress 2014**

**Scientific Progress and Accomplishments**

The overarching goals of this research project have been to improve the fidelity of quantum control in disparate hardware systems and to produce a deployable technology-independent quantum control toolkit. The work has been structured around a combination of efforts in hardware system design and improvement as well as “software” or protocols permitting improved quantum control fidelity. Taken together, and leveraging tradeoffs between the two areas where practical, these approaches have led to dramatic improvements in both achieved quantum control metrics and our understanding of how to effectively and efficiently implement error-resilient control.

**Experimental Quantum Control Capabilities:**

This project has yielded significant advances in the control of quantum coherent systems using both “AC” and “DC” control modalities. In particular, we highlight the following achievements as measured against our original proposal’s plan for 10-100X improvement in key coherence and fidelity metrics:

Trapped ions – T2 improved ~30x to > 3s via phase stabilization of 12.6 GHz reference used in our control system. Operational fidelity improved >10x to >99.99% via phase and amplitude stabilization of microwave synthesis and delivery system.

Key improvements have related to the implementation of a traceable timing synthesis chain linked to a Cs reference contributing to Universal Coordinated Time, an ultra-low-phase noise cleanup crystal oscillator, and low-phase-noise microwave components. We note that previous work on microwave components and systems in related programs have focused on noise metrics exclusively in the amplitude quadrature. These improvements have been vital as the phase stability of our selected trapped-ion platform means that the achievable coherence time is actually limited by the quality of our master-oscillator, rather than the quantum system itself. This is a key observation of value to the entire community: As measured coherence times increase in quantum systems the importance of deploying ultra-high-performance frequency references for quantum control hardware will increase in importance.

Singlet-Triplet qubits – T2* improved ~30x to >3us via stabilization of nuclear field gradients and implementation of real-time feedback corrections for drifting nuclear field gradient. Operational fidelity increased to >98% via pulse precompensation and nuclear field stabilization/estimation.

Key improvements have linked to improved estimation and compensation for drifting nuclear magnetic field gradients. Averaged over a temporal ensemble of measurements drifts in the nuclear field gradient between the two quantum dots comprising the physical qubit produce reduced visibility of Ramsey fringes (each experiment in the ensemble has a slightly different Ramsey period). The most recent development in this project has been the implementation of a Bayesian parameter estimation protocol which performs multiple measurements to estimate the nuclear field gradient and perform real-time corrections to the control system in response to this measurement. Effectively, this amounts to a form of automated real-time control system calibration, which has led to dramatic improvements in the measured coherence time for this system. The combination of

hardware and control software (in the form of Bayesian parameter estimation) are a powerful demonstration of fundamental structure of this research project.

**Noise Engineering:**

In addition to these direct improvements in our control systems we have also developed a new set of hardware-based tools for quantitative studies of quantum control in the form of noise engineering. We developed and demonstrated a technique to engineer universal unitary baths in quantum systems. Using the correspondence between unitary decoherence due to ambient environmental noise and errors in a control system for quantum bits, we showed how a wide variety of relevant classical error models may be realized through in-phase or in-quadrature modulation on a vector signal generator producing a resonant carrier signal. We demonstrated our approach through high-bandwidth modulation of the 12.6-GHz carrier appropriate for trapped Yb171+ ions. Experiments demonstrated the reduction of coherent lifetime in the system in the presence of both engineered dephasing noise during free evolution and engineered amplitude noise during driven operations. In both cases, the observed reduction of coherent lifetimes matched well with quantitative models described herein. These techniques form the basis of a toolkit for quantitative tests of quantum control protocols, helping experimentalists characterize the performance of their quantum coherent systems.

Generalized filter transfer functions for time-dependent noise:

Despite the ubiquity of decoherence in quantum information settings, it is a challenging problem to predict the average evolution of a qubit state undergoing a specific, but arbitrary operation in the presence of realistic time-dependent noise -- how much randomization does one expect and how well can one perform the target operation? Making such predictions accurately is precisely the capability that experimentalists require in realistic laboratory settings. Moreover, this capability is fundamental to the development of novel control techniques designed to modify or suppress decoherence as researchers attempt to build quantum-enabled technologies for applications such as quantum information and quantum sensing.

A key theoretical construct developed during this effort is the filter-transfer-function formalism, allowing the spectral properties of arbitrary quantum control operations to be calculated analytically. In this framework, the challenge of understanding the noise susceptibility of any control operation is simplified to calculating a spectral overlap integral between the analytic filter function and the noise power spectrum. This framework has yielded many novel theoretical insights published during the course of this program (e.g. Designing a practical high-fidelity long-time quantum memory, Nature Communications 2013), including extension to a multiqubit framework accounting for spatial correlations in the noise. Understanding the impact of correlations in this way is of fundamental importance to addressing questions of fault tolerance.

The FF description of ensemble-average quantum dynamics tremendously simplifies the task of analyzing the expected performance of a control protocol in a noisy environment as it permits consideration of control as noise spectral filtering. The FFs themselves may be described using familiar concepts such as frequency passbands, stopbands, and filter order, enabling a simple graphical representation of otherwise complex concepts in the dynamics of controlled quantum systems. Noise filtering, in practice, is achieved through construction of a control protocol which

modifies the controllability of the quantum system by the noisy environment over a defined frequency band. Adjusting the FF and changing its overlap with the noise spectrum thus allows a user to change the average dynamics of the system in a predictable way.

The intuitive nature of this framework is belied by the challenge of calculating FFs for arbitrary control protocols, generally involving time-domain modulation of control parameters such as the frequency and amplitude of a driving field. The nature of quantum dynamics means that the implemented control framework is generally nonlinear; for instance, one finds complex dynamics in circumstances where the noise and control operations do not commute such as a driven operation in the presence of dephasing noise. Our recent theoretical effort has allowed calculation of FFs for arbitrary single-qubit control and arbitrary universal classical noise, expanding significantly beyond previous demonstrations restricted to the identity operator in pure-dephasing environments. It is this more general case where the impact of noise filtering and the FFs may have the most significant impact on the quantum engineering community, and where experimental tests are vital.

Using capabilities in high-fidelity quantum control with trapped ions and noise engineering we have performed a wide-ranging set of experiments demonstrating the power and utility of the filter function formalism. We have used a “quantum spectrum analysis” technique to map out the filter functions for complex composite pulsing sequences and demonstrated excellent agreement with analytic calculations using no free parameters. Tests included primitive single-qubit rotations as well as advanced control protocols derived from both the NMR and quantum information communities. Our results have provided a vital experimental validation of generalized filter-transfer functions. Experiments demonstrated the utility of these constructs for directly predicting the evolution of a quantum state in a realistic noisy environment as well as for developing novel robust control protocols.

A new understanding of composite pulses for quantum control

Composite pulse (CP) sequences have long been employed in nuclear magnetic resonance (NMR) to mitigate the effects of systematic errors in the control. Initially developed to tackle static but otherwise unknown errors in the amplitude or frequency of the driving field, CPs are expressed as a composition of fundamental “primitive” rotations. CPs have recently been extended to handle multiple error sources using symmetry and concatenation and to provide efficient high-order error suppression by optimized design.

Despite these advances, an outstanding challenge to the systematic incorporation of CPs into practical quantum information systems has been a limited understanding of CP performance in the presence of realistic time-dependent noise. This is in contrast to e.g. optimal control approaches for gate synthesis, where the presence of time-dependent noise is typically assumed in the control design.

Treating the influence of time-dependent noise processes on quantum control operations beyond the limited examples appearing thus far in the literature has been facilitated by recent advances in dynamical error suppression based on open-loop Hamiltonian engineering. These approaches provide a general framework for

understanding and mitigating non-Markovian time-dependent noise in a finite-dimensional open quantum system due to either uncontrolled couplings to the environment or a variety of control errors, and have been central to the approaches studied in this research effort.

We have understood that approaches including both dynamical decoupling and dynamically corrected gates (DCGs) are able to perturbatively reduce the effects of classical as well as quantum noise sources, provided that the correlation time scale of the noise is sufficiently long compared to the control time scale at which the noise is "coherently averaged out." These characteristics may be captured quantitatively in filter transfer functions (FFs) for arbitrary single-qubit control using methods of spectral overlap in the frequency domain as developed as the centerpiece of this research program.

Via application of the FF formalism to study of CP performance, we demonstrated that CPs are able to effectively suppress control errors caused by time-dependent processes possessing realistic noise power spectra. Remarkably, robust performance of CP sequences is found up to fluctuations as fast as $\sim$10% of the Rabi frequency, providing an explicit quantitative characterization of the sensitivity of these approaches to time-dependent control noise. Calculations showed that even under such noise environments, which are beyond the static ones originally assumed for CPs, predicted fidelities are at least comparable to those for DCGs in scenarios where protocols of both kind are applicable. Altogether, our results show that, in combination, CP and DCG protocols provide experimentalists with a viable toolkit capable of meeting a variety of constraints, including the presence of colored time-dependent control noise.

Beyond examining the analytic properties of the filter functions calculated for CPs, we have deployed our experimental capabilities to reveal dramatic new insights into their performance. We experimentally demonstrated a form of quantum system identification, effectively reconstructing the amplitude-noise filter functions, for two well known compensating pulse sequences known by the shorthand designations SK1 and BB1. Calculations of fidelity using the engineering noise and analytically calculated FF match data well over the entire band in the weak noise limit with no free parameters.

Our choice of characterizing these compensating pulse sequences highlighted an important issue in the prediction of ensemble-average dynamics of controlled quantum systems. Ultimately, the underlying physical principles giving rise to the analytic form of the FFs are based on the well tested average Hamiltonian theory exploited in crafting these pulses. Despite this shared theoretical foundation, the calculation of spectral filtering properties is quite distinct from calculation of quasi-static error terms in a Magnus expansion, with important consequences for average quantum dynamics in realistic time-varying noise environments.

Accordingly, our tests of the FF formalism revealed that compensating pulses designed to suppress errors to high order in a Magnus-expansion framework needed not be efficient noise spectral filters. Despite significant differences in their construction -- the BB1 protocol is designed to provide higher-order cancellation of Magnus terms than SK1 -- both of the selected composite pulses provide similar filtering of time-dependent noise, given by the filter order (slope of the FF near zero frequency). In the weak-noise limit frequency-domain characteristics are captured accurately through the FF across frequencies ranging from quasi-static to rapidly fluctuating on the timescale of the pulse. Performance deviations between the pulses arose and the FF approximation broke down

as the noise strength is increased and higher order terms in the Magnus series became important, but only at low frequencies.  At frequencies fast relative to the control the FF again accurately predicted the relevant quantum dynamics even in the strong-noise limit.

These measurements were the first direct manifestation of the difference between studying quantum dynamics in terms of frequency-domain noise filtering and calculation of error contributions in a Magnus expansion as is appropriate in the quasi-static limit.  Accordingly we believe they have significant bearing on researchers interested in constructing error-suppressing protocols for quantum information systems subject to realistic time-dependent noise.

**Synthesis of noise filters for quantum control**

A range of techniques relying on both open- and closed-loop control have been devised to address the challenge of decoherence suppression at various levels in a layered architecture for quantum computing.  In particular, open-loop dynamical error suppression strategies have emerged as resource efficient approaches for physical-layer decoherence control.

For such schemes to be of realistic utility in scalable quantum information settings, however, a straightforward framework for selecting an error-suppressing protocol is required.   The key classes of bounded-strength SU(2) operations performing nontrivial single-qubit logic operations -- error-compensating composite pulse sequences in NMR and DCGs in quantum information -- have been crafted in an ad hoc manner, meeting contemporary needs of their respective disciplines of origin and functioning under different error models.  Moreover these schemes largely ignore realistic constraints imposed by the hardware that will be required for gate implementation such as finite timing precision and limited classical communication bandwidth between the physical (quantum) layer and any external classical controller.

A unified and experimentally relevant framework for the realization of error-suppressing gates was therefore needed to secure the role of dynamical error suppression in systematic designs of quantum technologies including fault-tolerant quantum computers.  Our motivation focused on leveraging insights from functional analysis in order to realize high-fidelity quantum control operations, and identified the Walsh functions - square-wave analogues of the sines and cosines - as natural building blocks for such protocols. The Walsh functions are defined in a uniform piecewise-constant fashion, building intrinsic compatibility with discrete clockingand classical digital logic, and have previously been identified as providing a unified mathematical framework in the context of quantum control sequencing.  We examined a Walsh-modulated driven qubit system weakly interacting with both dephasing and coherent amplitude-damping noise processes.

Control protocols could be constructed and completely characterized by their Walsh spectra, facilitating intuitive analytic design rules based on symmetry and spectral properties of the Walsh basis and the ability to perform Fourier-like synthesis using techniques well established in digital signal processing.  Modulation protocols were tailored to a particular operation on SU(2), and realized through study of effective filtering of time-dependent noise, as captured through the experimentally validated, generalized-filter-transfer-function formalism. We defined (and minimize) a cost-function effectively measuring the extent to which noise (over a user-defined spectral band) is filtered by the applied modulation.  We derived novel families of Walsh-modulated noise filters designed to suppress dephasing and coherent amplitude damping noise.

Experimental measurements confirmed our filter-synthesis framework, revealing that Walsh spectra predicted to yield high-order filters indeed provided robust noise suppression, and that they outperformed the best available primitive gates in the small error limit (demonstrated via randomized benchmarking). This suite of experiments is the capstone to our program, revealing the power and utility of our theoretical FF framework and setting the stage for a variety of novel control techniques to be developed in the future. Results are currently in press, and contribute to further manuscripts in preparation.

**Extension of DD and quantum memory beyond single qubits**

A major advance enabled by this grant during Year 2 has been the identification of single-qubit control protocols, based on periodic repetition of suitable high-order DD sequences, that can guarantee high fidelity at long storage times in the presence of Gaussian dephasing noise – while also meeting the practical constraint of low access latency. While this approach may be extended straightforwardly to designing a multi-qubit memory in the presence of independent Gaussian phase noise, it is not a priori clear whether more general dephasing models may be viable and, in particular, be amenable to support a “fidelity plateau” at long times – including non-Gaussian dephasing on a single qubit, and multi-qubit dephasing with spatial correlations. Likewise, although general-purpose multi-qubit DD sequences exist for decoherence suppression in the short-time regime (notably, multi-qubit Concatenated DD, CDD, or so-called Nested Uhrig DD, NUDD), it is not a priori clear whether more resource-efficient pulse sequences may be built by specifically taking into account the purely-dephasing nature of the error model.

Working together with postdoctoral fellow Gerardo Paz-Silva, as well as (former) postdoc Seung-Woo Lee and visiting student Todd Green (from Biercuk’s group), Viola has tackled the above issues for arbitrary classical and quantum multi-qubit dephasing settings, by considering access to both “non-selective” (global) and “selective” control capabilities. Throughout the analysis, a key technical tool was provided by a suitable generalization of the FF approach to multi-qubit systems, capturing in general both the decay and the non-trivial phase evolution that the coupling to a correlated dephasing environment may induce. Remarkably, provided that qubit-selective control pulses are available, it turned out that demanding suitable displacement symmetry conditions on the sequence propagator, one may simultaneously: (i) achieve an exponential reduction in the required number of pulses for high-order DD, as compared to standard NUDD schemes; (ii) ensure that, if appropriate time-scale and spectral conditions are met, a plateau behavior may still be engineered for each coherence order (hence for fidelity). Further to that, these enhanced DD sequences exhibit the additional advantage of intrinsic model robustness, in the sense that their performance is insensitive to the specific (possibly unknown) details of the environmental coupling – in particular, fully collective vs. independent dephasing. These findings are reported in an extended manuscript which is presently in the final stages of writing: “Dynamical decoupling sequences for multi-qubit dephasing suppression and long-time quantum memory,” by G. Paz-Silva, S.-W. Lee, T. J. Green, and L. Viola, in preparation for New Journal of Physics (expected submission: October 2014)

On the singlet-triplet qubits we have experemintally implemented a two qubit DD sequence and used it to generate controlled entanglement of two ST qubits (Science 2012).

**Noise Suppresion via Hamiltonian Estimation**

Unwanted interaction between a quantum system and its fluctuating environment leads to decoherence and is the primary obstacle to establishing a scalable quantum information processing architecture. Strategies such as environmental and materials engineering, quantum error correction and DD can mitigate decoherence, but generally increase experimental complexity. During the last year of the project we improved coherence in a ST qubit using real-time Hamiltonian parameter

estimation. Using a rapidly converging Bayesian approach, we precisely measure the splitting in a singlet-triplet spin qubit faster than the surrounding nuclear bath fluctuates. We continuously adjust qubit control parameters based on this information, thereby improving the inhomogenously broadened coherence time ($T2^*$ ) from tens of nanoseconds to above 2 µs and demonstrating the effectiveness of Hamiltonian estimation in reducing the effects of correlated noise in quantum systems. Because the technique demonstrated here is compatible with arbitrary qubit operations, it is

a natural complement to quantum error correction and can be used to improve the performance of a wide variety of qubits in both metrological and quantum-information-processing applications.